

# DETECTION OF MASSES IN MAMMOGRAPHY THROUGH REDUNDANT EXPANSIONS OF SCALE

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**Abstract**—We show that dyadic scales may not be sufficient for the detection of masses in mammograms: a lesion may be too blurred on one scale, and then too fragmented at the next. In this paper, we report on the preliminary evidence of our study using a continuous wavelet transform in two dimensions with arbitrary positioning of a wavelet's center frequency channel tuned to the mass detection problem. Our goal is to detect masses in dense mammograms whose diameter is smaller than 1 cm. The aim is to be able to find the scale where the mass is best represented in terms of analysis.

**Keywords** – Mammography, wavelet, expansion, mass detection, scale.

## I. INTRODUCTION

An initial study in one dimension helped us observe that dyadic scales are often not sufficient to detect a mass in a dense mammogram [3]. Below we show this by a continuous wavelet transform, which computes the decomposition on voices between traditional dyadic scales.

## II. METHODOLOGY

1) *Voices and octaves*: It is possible to expand a signal more finely and compute scales between octaves of traditional dyadic expansions by voices [1]. A voice constitutes a subdivision of an octave. If we consider a wavelet mother  $\psi$ , a family of wavelets

$\psi_{m,n}(x) = a_0^{-\frac{m}{2}} \psi(a_0^{-m}x - nb_0)$  where  $a_0$  is the dilatation parameter,

$b_0$  is the translation parameter,  $(m, n) \in \mathbf{Z}^2$  are possible. In

the dyadic case,  $a_0=2$  and  $b_0=1$ ,  $\psi_{m,n}(x) = 2^{-\frac{m}{2}} \psi(2^{-m}x - n)$ .

Decomposing N voices per octave means creating N functions

$\psi_{n,m}^v$  and computing the frame

$\{\psi_{m,n}^v : (m, n) \in \mathbf{Z}^2, v = 1, \dots, N\}$ . Analyzing with N

voices means finding N different frequency channels, which

correspond to the N frequency localizations of  $y^1, \dots, y^N$  [2],

all translated by the same step (Fig. 1b). Such a lattice can be

viewed as the superposition of N different lattices of the type shown in Fig. 1a, stretched by fixed amounts in frequency.

For example a possible choice for  $y^n$  is

$$\psi^v(x) = 2^{-\frac{v-1}{N}} \psi(2^{-\frac{v-1}{N}}x).$$

If  $|\hat{\psi}(\xi)|$ , which we assume to be even, peaks around  $\pm\omega_0$ ,

then  $|\hat{\psi}^j|$  will be concentrated around  $\pm 2^{\frac{j-1}{N}} \omega_0$  in the same

way as in the dyadic case. If  $|\hat{y}|$  has two peaks in frequency

at  $\pm x_0$ ,  $|\hat{y}_{m,n}(x)|$  then peaks at  $\pm 2^m x_0$  which are two localization centers of  $\psi_{m,n}$ .

The equation computing the scale for source given “octave”, “current voice” and “number of voices” is

$$scale = 2^{\frac{octave + \frac{current\_voice}{number\_voices}}{3}} [3].$$

Moreover, we adopt the following convention: the first octave (octave number zero) corresponds to the width between scales

1 +  $\frac{1}{number\_voices}$  and 2. The dyadic scale of an octave is the last voice of the octave (scale =  $2^{octave+1}$ ). In Fig. 2, we consider a signal of 512 points ( $2^9$ ). This means 9 octaves (octave 0 to octave 8). The coarsest scale is 512, the finest is

1 +  $\frac{1}{number\_voices}$ . For example, when we display a second

voice of the fourth octave (four voices per octave computed),

we obtain the scale  $2^{4+\frac{2}{4}}$ , that is to say scale 23.

2) *One dimensional experiment*: We applied programs from libraries in LastWave and Matlab, using a continuous wavelet transform and a discrete wavelet transform. LastWave is a wavelet signal and image-processing environment, written in C [4]. Wavelab is an extension of Matlab. For the CWT, we concentrated on the first and the second derivative of a gaussian function (Mexican Hat wavelet). We processed phantom signals with three masses of distinct sizes using gaussian additive noise.

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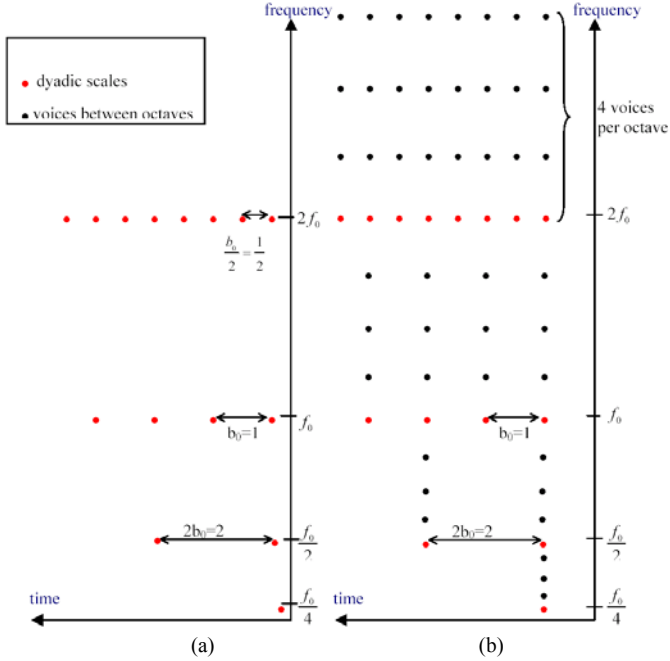


Fig. 1. The time-frequency lattice: (a) for the dyadic wavelet transform,  $\psi_{m,n}$  is localized around  $2^m n b_0$ ;  $a_0=2$  and we assume  $b_0=1$ ; (b) for a scheme with four voices, the different voice wavelets  $\psi^1, \dots, \psi^4$  are assumed to be dilations of a single function  $\psi$ ,  $\psi^j(x) = 2^{-\frac{j-1}{4}} \psi(2^{-\frac{j-1}{4}} x)$ .

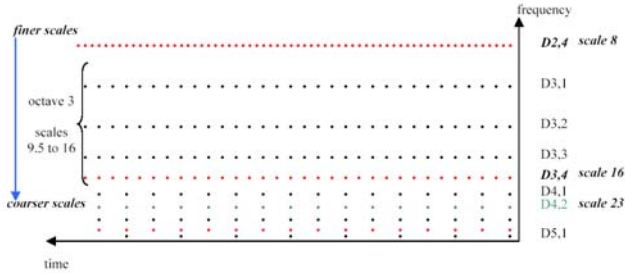


Fig. 2. The time-frequency lattice for a scheme with four voices per octave, including the scale axis.

We plotted two scan line profiles (Fig. 4) of a real mammogram (Fig. 3).

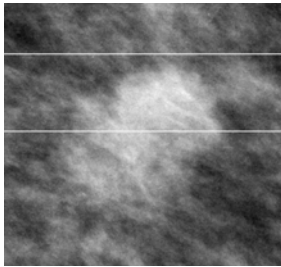


Fig. 3. Real mass from a mammogram. The white lines show the locations of the extracted profiles corresponding to Fig. 4.

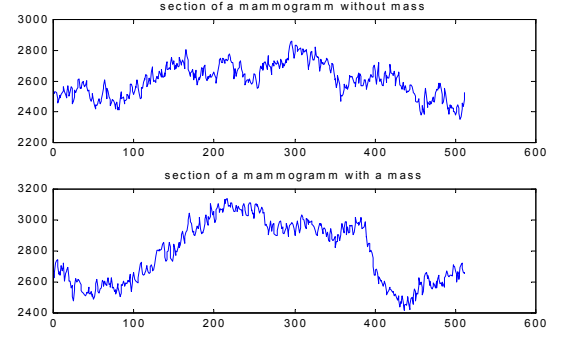


Fig. 4. 1D sections of a real mammogram.

As shown in Fig. 4, we added gaussian noise on the phantom mass so that the 1D signal had approximately the same shape as a real mass. Fig. 5 shows this representation.

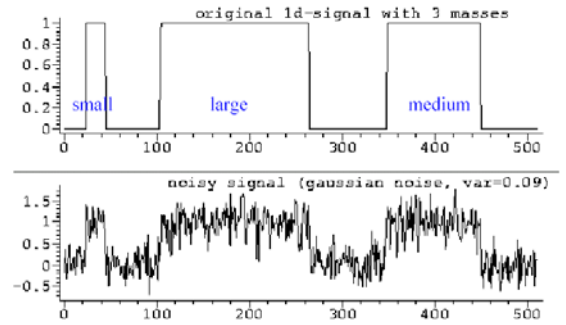


Fig. 5. Phantom signal with an added white gaussian noise of variance 0.1.

Fig. 6 depicts the results obtained without downsampling. The signal was composed of masses with a white gaussian noise of variance 0.1. The wavelet was a Mexican Hat.

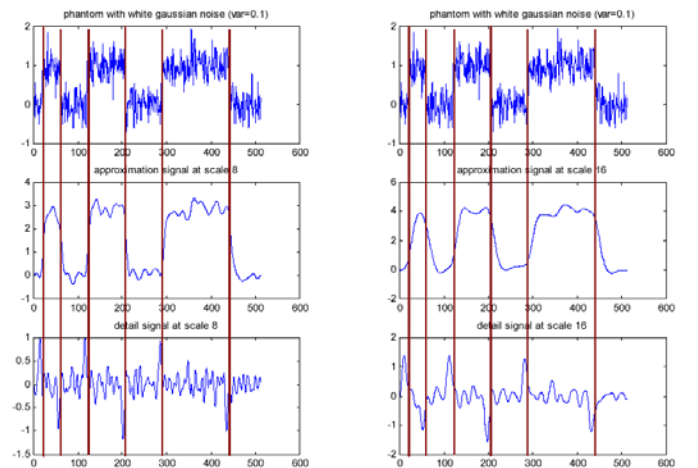


Fig. 6. Analysis by a DWT on a phantom with gaussian noise (var = 0.1). We show approximation and detail signals at scales 8 (left) and 16 (right).

3) *The 2D CWT*: We began the 2D study with phantom masses of white objects on a black background with the addition of white gaussian noise of variance 4. We applied a bias to the magnitude values to preserve the waveform shape and make the signal purely positive [5]. Next, we performed the analysis on a cancerous mass from a mammogram (Fig. 3). We show the biased unthresholded results in Fig. 7, and the thresholded values in Fig. 8.

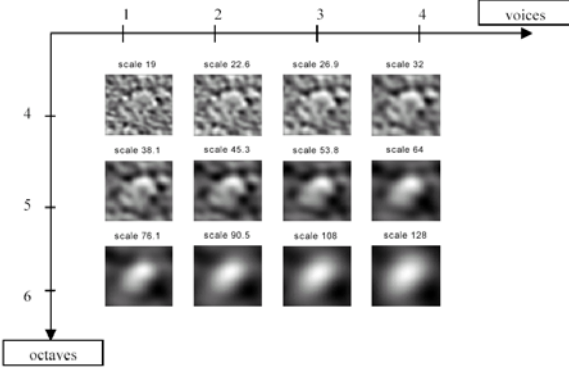


Fig. 7. CWT\_2D at octaves 4 to 6, four voices per octave. No thresholding on biased coefficients.

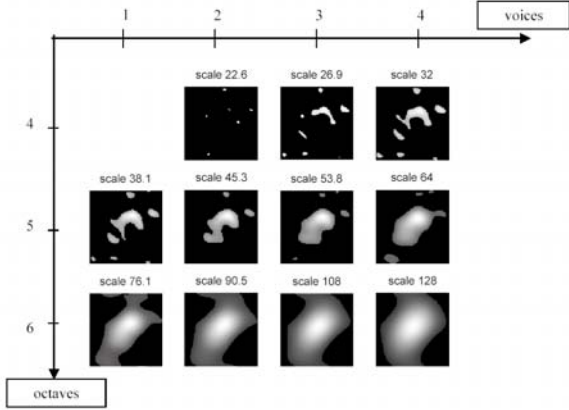


Fig. 8. CWT\_2D at octaves 4, 5 and 6, four voices per octave. Coefficients are biased and thresholded among scales (10 for scale 38 to 20 for scale 128).

4) *Fractional Splines*: We have more recently focused on the Fractional Spline Wavelet Transform [6,7]. We have extended the implementation to two dimensions, which was described originally by M. Unser and T. Blu [8]. We used orthonormal filters to compute the details (horizontal, vertical and diagonal) and the approximation coefficients of the image by applying the filters,

$$H_{\perp}^{\alpha}(e^{j\omega}) = \sqrt{2} \left( \frac{1 + e^{-j\omega}}{2} \right)^{\alpha+1} \sqrt{\frac{A^{2\alpha+1}(e^{j\omega})}{A^{2\alpha+1}(e^{2j\omega})}}$$

$$G_{\perp}^{\alpha}(e^{j\omega}) = e^{-j\omega} H_{\perp}^{\alpha}(-e^{-j\omega})$$

where  $A^{\alpha}(z)$  is the autocorrelation filter of degree  $\alpha$ .

The transform is computed for a real mass along the scales for different values of the spline parameter  $\alpha$  (Fig. 9).

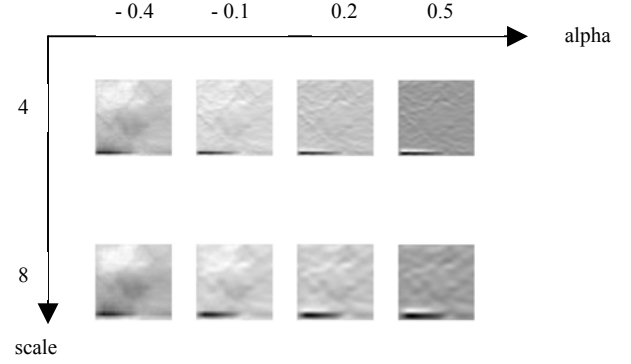


Fig. 9. DWT in 2D at scale 4 and 8, for 4 values of  $\alpha$ .

As shown in Fig. 9, we do not always observe a good representation for different values of the parameter  $\alpha$ . However, we clearly observe that the detection is better for  $\alpha=0.2$ . The parameter of the spline is continuous ( $\alpha > -0.5$ ). Therefore, it is interesting to make the parameter vary in order to find the best basis, which suits well a given mass size. However the present transform is only computed at dyadic scales. With a continuous analysis, which would allow decomposition on voices between these scales, we may obtain a richer parameter space so as to identify a best basis for mass detection.

### III. DISCUSSION

Given the results in one dimension, we then implemented a 2D continuous wavelet transform. Our goal was to now find the most suitable scale to detect a mass of arbitrary size. To find the best scale, we displayed the maxima of the coefficients along scales, the third dimension giving the magnitude of the maxima at each scale. In addition, we plotted the correlation between the original mass and the coefficients of the CWT at each scale. We expected to find different optimal scales according to the size of a mass. We tested this by carrying out our algorithm on three different size masses. We first computed for each mass the CWT in 2D on 9 possible octaves (3 voices per octave). Then for each octave and scale we plotted the maxima of the coefficients of the wavelet decomposition as shown in Fig. 10.

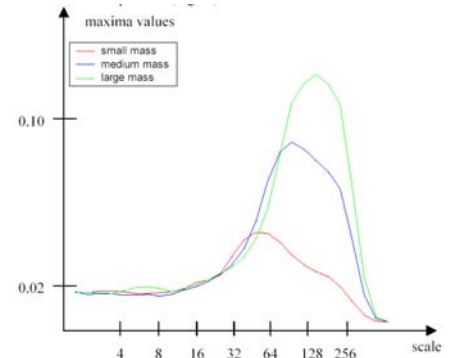


Fig. 10. Evolution of the maxima of the cwt2d across scales.

The positions of the maxima of the decomposition were at scales 40, 81 and 128 for small, medium and large masses respectively.

Next, we performed the CWT in 2D on the same number of octaves and voices. For each scale we calculated the correlation between the original image without noise and the 2D CWT decomposition as shown in Fig. 11.

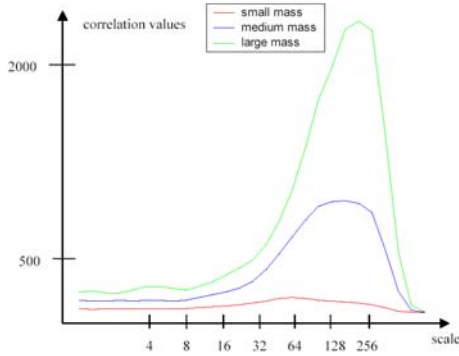


Fig. 11. Correlation between the original image and the biased values of the decomposition.

The positions of the maxima of the correlation were at scales 64, 102 and 161 for small, medium and large masses respectively.

The most suitable scale using the method of the maxima evolution was not the same as the scale identified with correlation. Next, we attempted to find the best scale for a real mass. This time, the best scale to detect the real mass was the same for both methods (maxima evolution and correlation) at scale 161.

We also considered a very noisy signal (variance 4), for robustness. We analyzed the maxima of the coefficients and the correlation for different noise settings. From these results we observed that both methods identified same scale values regardless of the amount of added noise.

#### IV. CONCLUSION

Our studies in one and two dimensions suggest that dyadic scales are often not sufficient to detect a mass in a dense mammogram. We showed the advantage of a continuous wavelet transform, which computed an expansion on voices between the common dyadic scales. We saw on real images of masses extracted from digitized mammograms that a correlation method between a known mass and the values of computed coefficients yielded approximately the same results, as a maximum method evolution. Thus, this study suggests that it is possible and of value to tune an analysis between octaves, for the detection of subtle masses in mammograms.

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